# Sample Question Paper - 41
# Mathematics-Standard (041)
# Class- X, Session: 2021-22
# TERM II

***Time Allowed : 2 hours*** ***Maximum Marks : 40***

## *General Instructions :*

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

## SECTION - A

**1.** Solve for $x$ : $x^2 - (2b - 1)x + (b^2 - b - 20) = 0$

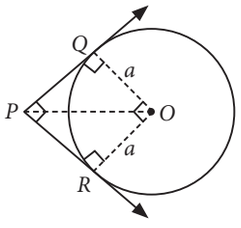
**2.** The angles of a quadrilateral are in A.P. whose common difference is 10°. Find the angles.

**OR**

The sum of the 2nd and the 7th term of an A.P. is 30. If its 15th term is 1 less than twice its 8th term, then find the A.P.

**3.** If $ad \neq bc$, then prove that the equation $(a^2 + b^2)x^2 + 2(ac + bd)\,x + (c^2 + d^2) = 0$ has no real roots.

**4.** If angle between two tangents drawn from a point $P$ to a circle of radius '$a$' and centre $O$ is 90°, then find the length of $OP$.




**5.** The length of a cold storage is double its breadth. Its height is 3 metres. The areas of its four walls (including doors) is 108 $m^2$. Find its volume.

**OR**

The radii of the internal and external surfaces of a metallic spherical shell are 3 cm and 5 cm respectively. It is melted and recut into a solid right circular cylinder of height $10\frac{2}{3}$ cm. Find the diameter of the base of the cylinder.

**6.** If the mode of the given data is 340, find the missing frequency $x$ for the following data :

| Classes | 0-100 | 100-200 | 200-300 | 300-400 | 400-500 | 500-600 |
|---|---|---|---|---|---|---|
| Frequency | 8 | 12 | $x$ | 20 | 14 | 7 |

## SECTION - B

**7.** The following table gives the literacy rate (in %) in 40 cities. Find the mean literacy rate.

| Literacy rate (in %) | 45-55 | 55-65 | 65-75 | 75-85 | 85-95 |
|---|---|---|---|---|---|
| Number of cities | 4 | 11 | 12 | 9 | 4 |

**8.** The angle of elevation of a cloud from a point 60 m above the surface of the water of a lake is 30° and the angle of depression of its shadow in water of lake is 60°. Find the height of the cloud from the surface of water.

**OR**

From a point $P$ on the ground, the angle of elevation of the top of a 10 m tall building is 30°. A flagstaff is fixed at the top of the building and the angle of elevation of the top of the flagstaff from $P$ is 45°. Find the length of the flagstaff and the distance of the building from the point $P$. (Take $\sqrt{3} = 1.73$)

**9.** In an apple orchard, the number of apples on 80 trees are as follows :

| Number of apples | 40-60 | 60-80 | 80-100 | 100-120 | 120-140 | 140-160 | 160-180 |
|---|---|---|---|---|---|---|---|
| Number of trees | 12 | 11 | 14 | 16 | 13 | 9 | 5 |

Find the median of the above data.

**10.** Construct a right triangle $ABC$ with $AB = 6$ cm, $BC = 8$ cm and $\angle B = 90°$. Draw $BD$, the perpendicular from $B$ on $AC$. Draw the circle through $B$, $C$ and $D$ and construct the tangents from $A$ to this circle.

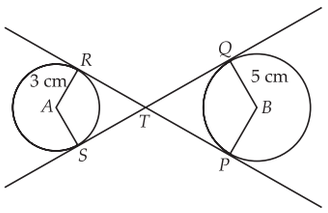
## SECTION - C

**11.** A conical vessel of radius 12 cm and height 16 cm is completely filled with water. A sphere is lowered into the water and its size is such that, when it touches the sides, it is just immersed. What fraction of the water overflows?

**12.** $AB$ and $CD$ are two parallel chords of a circle such that $AB$ = 10 cm and $CD$ = 24 cm. The chords are on opposite sides of the centre and the distance between them is 17 cm. Find the radius of the circle.

**OR**

In the fig, $RTP$ and $STQ$ are common tangents to the two circles with centres $A$ and $B$. The radii of the two circles are 3 cm and 5 cm respectively. If $ST : TQ = 1 : 3$ and $RT = 4$ cm. Find the length of $QT$ and $AB$.



## Case Study - 1

**13.** Amit is preparing for his upcoming semester exam. For this, he has to practice the chapter of Quadratic Equations. So he started with factorization method. Let two linear factors of $ax^2 + bx + c$ be $(px + q)$ and $(rx + s)$.

$\therefore \quad ax^2 + bx + c = (px + q)(rx + s) = prx^2 + (ps + qr)x + qs.$

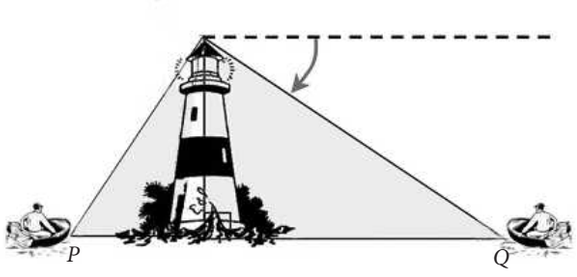
Now, factorize each of the following quadratic equations and find the roots.

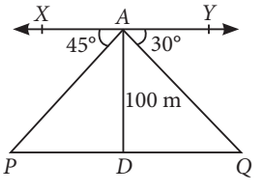
(i) $6x^2 + x - 2 = 0$

(ii) $x^2 - 28x + 160 = 0$

## Case Study - 2

**14.** A boy is standing on the top of light house. He observed that boat $P$ and boat $Q$ are approaching to light house from opposite directions. He finds that angle of depression of boat $P$ is 45° and angle of depression of boat $Q$ is 30°. He also knows that height of the light house is 100 m.




Based on the above information, answer the following questions.

(i) Find the length of $PD$.

(ii) Find the length of $DQ$.

## Solution

## MATHEMATICS STANDARD 041

## Class 10 - Mathematics

**1.** We have, $x^2 - (2b - 1)x + (b^2 - b - 20) = 0$
Discriminant, $D = (2b - 1)^2 - 4(1)(b^2 - b - 20)$
$= 4b^2 + 1 - 4b - 4b^2 + 4b + 80 = 81$
Using quadratic formula,

$$x = \frac{(2b-1) \pm \sqrt{81}}{2(1)} = \frac{2b-1 \pm 9}{2}$$

$$\Rightarrow \quad x = \frac{(2b-1)+9}{2} \text{ or } x = \frac{(2b-1)-9}{2}$$

$$\Rightarrow \quad x = \frac{2b+8}{2} = b+4 \quad \text{or} \quad x = \frac{2b-10}{2} = b-5$$

**2.** Let the four angles of a quadrilateral are $n$, $(n + 10°)$, $(n + 20°)$ and $(n + 30°)$.
$\because$ Sum of all the angles of a quadrilateral = 360°
$\Rightarrow n + (n + 10°) + (n + 20°) + (n + 30°) = 360°$
$\Rightarrow 4n + 60° = 360° \Rightarrow n = 300°/4 = 75°$
$\therefore$ Angles are 75°, 85°, 95° and 105°.

**OR**

Let $a$ be the first term and $d$ be the common difference of the A.P. Now, according to the question, $a_2 + a_7 = 30$
$\Rightarrow a + d + a + 6d = 30 \Rightarrow 2a + 7d = 30$ ...(i)
Given, $a_{15} = 2a_8 - 1$
$\Rightarrow a + 14d = 2(a + 7d) - 1 \Rightarrow a + 14d = 2a + 14d - 1$
$\Rightarrow a = 1$ ...(ii)
Substituting (ii) in (i), we get
$2 + 7d = 30 \Rightarrow 7d = 28 \Rightarrow d = 4$
Hence, the A.P. is formed as 1, 5, 9, ....

**3.** We have, $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$
Discriminant, $D = 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2)$
$= 4(a^2c^2 + b^2d^2 + 2acbd) - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$
$= 4(a^2c^2 + b^2d^2 + 2abcd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2)$
$= 4(2abcd - a^2d^2 - b^2c^2) = -4(ad - bc)^2 < 0 \quad [\because ad \neq bc]$
Thus, given equation has no real roots.

**4.** In given figure, $PQ$ and $PR$ be the tangents.
Since $\angle P = 90°$
$\therefore$ Using angle sum property in quad $OQPR$,
$\angle QOR = 90°$
Also, $OR = OQ = a$
$\therefore$ $PQOR$ is a square

$$\Rightarrow OP = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$$

**5.** Let the length, breadth and height of the cold storage be $l$, $b$ and $h$ metres respectively. Then $l = 2b$ (given) and $h = 3$ m.
Now, area of the four walls = 108 m$^2$
$\Rightarrow 2(l + b)h = 108$
$\Rightarrow 2(2b + b) \times 3 = 108$
$\Rightarrow 18b = 108 \Rightarrow b = 6$
$\Rightarrow l = 2 \times 6 = 12$ m
Hence, volume of the cold storage
$= l \times b \times h = 12 \times 6 \times 3 = 216$ m$^3$.

**OR**

Let the radius of the base of the cylinder be $r$ cm. Then,
Volume of the metallic solid cylinder of height $10\frac{2}{3}$ cm = Volume of the metal in the spherical shell

$$\Rightarrow \quad \pi \times r^2 \times \frac{32}{3} = \frac{4}{3}\pi(5^3 - 3^3)$$

$$\Rightarrow \quad \frac{32}{3}r^2 = \frac{4}{3}(125 - 27) \quad \Rightarrow \quad r^2 = \frac{3}{32} \times \frac{4}{3} \times 98$$

$$\Rightarrow \quad r^2 = \frac{49}{4} \Rightarrow r = \frac{7}{2} \text{ cm}$$

Hence, diameter of the base of the cylinder = 7 cm

**6.** Here, mode = 340 which lies in the interval 300-400.
$\therefore$ Modal class = 300-400
Now, Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$$\Rightarrow \quad 340 = 300 + \left(\frac{20 - x}{2 \times 20 - x - 14}\right) \times 100$$

$$\Rightarrow \quad 340 - 300 = \left(\frac{20 - x}{26 - x}\right) \times 100$$

$\Rightarrow 6x = 96 \Rightarrow x = 16$

**7.** The frequency distribution table from the given data can be drawn as :

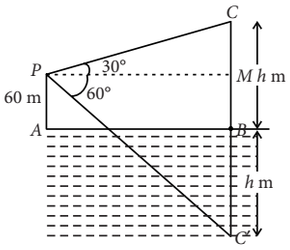
| Literacy rate | Class-marks ($x_i$) | Frequency ($f_i$) | $u_i = \frac{x_i - A}{h}$ | $f_iu_i$ |
|---|---|---|---|---|
| 45-55 | 50 | 4 | –2 | –8 |
| 55-65 | 60 | 11 | –1 | –11 |
| 65-75 | 70 | 12 | 0 | 0 |
| 75-85 | 80 | 9 | 1 | 9 |
| 85-95 | 90 | 4 | 2 | 8 |
| Total | | $\Sigma f_i = 40$ | | $\Sigma f_iu_i = -2$ |

Let assumed mean ($A$) = 70

$$\therefore \quad \text{Mean } (\bar{X}) = A + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$= 70 + \left(\frac{-2}{40}\right) \times 10 = 70 - \frac{20}{40} = 69.5$$

**8.** Let $AB$ be the surface of the lake and $C$ be the position of cloud and $C'$ be its reflection or shadow in the lake. Also, let height of cloud is $h$ m.




Here, $PM = AB$ and $BM = AP = 60$ m

In $\Delta PCM$, $\tan 30° = \frac{CM}{PM} = \frac{BC - BM}{AB}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h - 60}{AB}$

$\Rightarrow \quad AB = (h - 60)\sqrt{3}$ ...(i)

In $\Delta PMC'$, $\tan 60° = \frac{C'M}{PM} = \frac{BC' + BM}{AB}$

$\Rightarrow \quad \sqrt{3} = \frac{h + 60}{AB} \Rightarrow \quad AB = \frac{h + 60}{\sqrt{3}}$ ...(ii)

From (i) and (ii), we have

$(h - 60)\sqrt{3} = \frac{h + 60}{\sqrt{3}} \Rightarrow 3h - 180 = h + 60$

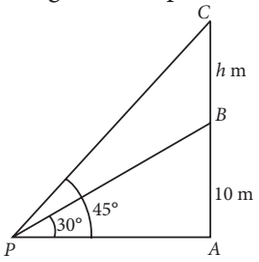
$\Rightarrow 2h = 240 \Rightarrow h = 120$

Thus, height of the cloud from the surface of water is 120 m.

**OR**

Let $AB$ be the building and $BC$ be the flagstaff of height $h$ m. $AP$ is the distance of the building from the point $P$.

In right $\Delta PAB$,




$\tan 30° = \frac{AB}{AP}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{10}{AP}$

$\Rightarrow \quad AP = 10\sqrt{3}$ m

$\Rightarrow AP = 10 \times 1.73$ m $= 17.3$ m ...(i)

So, the distance of the building from the point $P$ is 17.3 m.

In right $\Delta PAC$, $\tan 45° = \frac{AC}{AP}$

$\Rightarrow \quad 1 = \frac{10 + h}{17.3}$ (Using (i))

$\Rightarrow 10 + h = 17.3$

$\Rightarrow h = 17.3 - 10 \Rightarrow h = 7.3$

Thus, the length of the flagstaff is 7.3 m.

**9.** The frequency distribution table from the given data can be drawn as :

| Class | Frequency ($f_i$) | Cumulative frequency ($c.f.$) |
|---|---|---|
| 40-60 | 12 | 12 |
| 60-80 | 11 | 23 |
| 80-100 | 14 | 37 |
| 100-120 | 16 | 53 |
| 120-140 | 13 | 66 |
| 140-160 | 9 | 75 |
| 160-180 | 5 | 80 |
| Total | 80 | |

Clearly, $\frac{N}{2} = \frac{80}{2} = 40$ lies in the class interval 100-120.

So, 100-120 is the median class.

$\therefore \quad l = 100$, c.f. $= 37$, $f = 16$, $h = 20$

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h$$

$$= 100 + \left(\frac{40 - 37}{16}\right) \times 20$$

$$= 100 + \frac{60}{16} = 100 + 3.75 = 103.75$$

$\therefore$ Median $= 103.75$

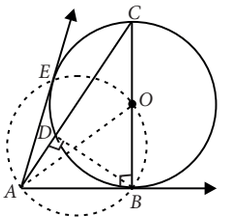
**10. Steps of construction :**

**Step-I :** Draw $\Delta ABC$ and perpendicular $BD$ from $B$ on $AC$.

**Step-II :** Draw a circle with $BC$ as diameter. This circle will pass through $D$.

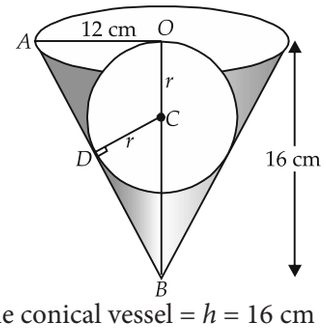
**Step-III :** Let $O$ be the mid-point of $BC$. Join $AO$.

**Step-IV :** Draw a circle with $AO$ as diameter. This circle cuts the circle drawn in step 2 at $B$ and $E$.




**Step-V :** Join $AE$. $AE$ and $AB$ are desired tangents drawn from $A$ to the circle passing through $B$, $C$ and $D$.

**11.** Radius of conical vessel = $R$ = 12 cm




Height of the conical vessel = $h$ = 16 cm
Let the radius of sphere = $r$
Now, $\Delta BOA \sim \Delta BDC$,

$$\Rightarrow \quad \frac{AO}{CD} = \frac{AB}{BC}$$

$$\Rightarrow \quad \frac{12}{r} = \frac{20}{16-r} \quad \left[\text{In } \Delta AOB, AB = \sqrt{16^2+12^2} = 20\right]$$

$$\Rightarrow \quad 12 \times (16 - r) = 20r \Rightarrow 32r = 16 \times 12$$

$$\Rightarrow \quad r = \frac{16 \times 12}{32} \text{ cm} = 6 \text{ cm}$$

Volume of water that overflows = Volume of the sphere

$$= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7}(6)^3 = \frac{6336}{7} \text{ cm}^3$$

Volume of water in conical vessel $= \frac{1}{3}\pi R^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (12)^2 \times 16 = \frac{16896}{7}$$

Fraction of water that over flows

$$= \frac{\text{Volume of water overflows}}{\text{Volume of water in conical vessel}}$$

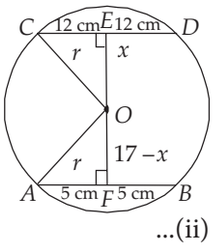
$$= \frac{6336}{7} \times \frac{7}{16896} = \frac{3}{8}$$

**12.** Given, $AB$ = 10 cm, $CD$ = 24 cm
$EF$ (distance between two chords) = 17 cm
Let $OE = x$, then $OF = EF - OE = 17 - x$
and $OC = OA = r$ [Radii of the circle]
In $\Delta OCE$, right angled at $E$

$$OC^2 = CE^2 + OE^2$$

$$\Rightarrow \quad r^2 = 12^2 + x^2$$

$$\Rightarrow \quad r^2 = 144 + x^2 \qquad \ldots(i)$$




In $\Delta OAF$, right angled at $F$

$$OA^2 = AF^2 + OF^2$$

$$\Rightarrow \quad r^2 = 5^2 + (17 - x)^2$$

$$\Rightarrow \quad r^2 = 25 + 289 - 34x + x^2 \qquad \ldots(ii)$$

From (i) and (ii), we get

$$144 + x^2 = 25 + 289 - 34x + x^2$$

$$\Rightarrow \quad 34x = 170 \Rightarrow x = 5 \text{ cm}$$

Now, from eq (i), we have

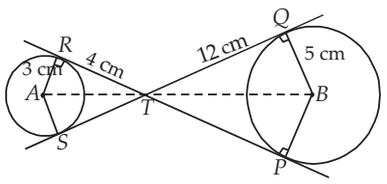
$$r^2 = 144 + 25 = 169$$

$$\Rightarrow \quad r = 13 \text{ cm}$$

Hence radius of the circle is 13 cm.

**OR**

We have, $AR$ = 3 cm, $BQ$ = 5 cm, $RT$ = 4 cm and $ST : TQ = 1 : 3$




$TR$ and $TS$ are two tangents drawn from an external point $T$ to the circle with centre $A$

$$\therefore \quad TR = TS \text{ and } \frac{ST}{TQ} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{TR}{TQ} = \frac{1}{3} \Rightarrow \frac{4}{TQ} = \frac{1}{3} \qquad [\because TR = 4 \text{ cm}]$$

$$\Rightarrow \quad TQ = 12 \text{ cm}$$

Now, in $\Delta ART$, $\angle R = 90°$

$$\therefore \quad AT^2 = AR^2 + RT^2$$

$$\Rightarrow \quad AT^2 = (3)^2 + (4)^2 = 9 + 16 = 25 \Rightarrow AT = 5 \text{ cm}$$

and in $\Delta BQT$, $\angle Q = 90°$

$$\therefore \quad BT^2 = BQ^2 + TQ^2$$

$$\Rightarrow \quad BT^2 = (5)^2 + (12)^2 = 25 + 144 = 169$$

$$\Rightarrow \quad BT = 13 \text{ cm}$$

Now $AB = AT + BT$

$$\Rightarrow \quad AB = 5 + 13 = 18 \text{ cm}$$

Hence, $QT$ = 12 cm and $AB$ = 18 cm

**13.** (i) We have, $6x^2 + x - 2 = 0$

$$\Rightarrow \quad 6x^2 - 3x + 4x - 2 = 0 \Rightarrow (3x + 2)(2x - 1) = 0$$

$$\Rightarrow \quad x = \frac{1}{2}, \frac{-2}{3}$$

(ii) $x^2 - 28x - 160 = 0$

$$\Rightarrow \quad x^2 - 20x - 8x + 160 = 0$$

$$\Rightarrow \quad x(x - 20) - 8(x - 20) = 0$$

$$\Rightarrow \quad (x - 20)(x - 8) = 0$$

$$\therefore \quad x = 20 \text{ or } 8.$$

**14.** (i) $\angle XAP = 45°$ (Given)

$$\therefore \quad \angle APD = 45° \qquad \text{[Alternate interior angles]}$$

In $\Delta APD$, $\frac{AD}{DP} = \tan 45°$

$$\Rightarrow \quad \frac{100}{DP} = 1 \Rightarrow DP = 100 \text{ m}$$

(ii) In $\Delta AQD$, $\frac{AD}{QD} = \tan 30°$

$$\Rightarrow \quad \frac{100}{QD} = \frac{1}{\sqrt{3}} \Rightarrow QD = 100\sqrt{3} \text{ m}$$